Superpositions of causality: an analysis of the mechanisms of control in the quantum limit

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- The roles of state preparation and measurement
- Quantum coherence as a necessary element of causality
- A constructive resolution of quantum puzzles

Where's the physics in "state vectors"?

Physics can only be explained by observable effects.

1. Quantum State $|\psi\rangle$:
   Summarizes the effects of external causes on the system

2. Measurement $|m\rangle$:
   Summarizes the causes of an external effect on the system

Quantum systems serve as conduits of causality between external conditions and their eventual effects.

Quantum states describe the universal relations between system dynamics and external effects.

Hilbert space vectors are associated with the dynamics of the system conditioned by external realities.

A simple theory of state preparation

State preparation can be represented by a *Filter operator* $\widehat{F}$ applied to an initially random state of the system,

$$\hat{\rho} = \frac{\widehat{F}\widehat{I}\widehat{F}^\dagger}{\mathrm{Tr}(\widehat{F}\widehat{I}\widehat{F}^\dagger)}$$

$\widehat{F}$ represents the means of control. It summarizes interactions and external effects registered outside the system.

Physically, the dynamics of the system cannot be separated from its external effects. Even a self-adjoint operation $\widehat{F}$ describes quantum coherent superpositions of unitary transformations acting on the system.

State preparation as ergodic randomization

Hofmann, Eur. Phys. J. D **70**, 118 (2016); Hibino et al., arXiv:1705.05118

Quantum states are characterized by uncertainties in the physical properties of a system. An interaction that reduces the uncertainty in a property $\hat{A}$ randomizes the dynamics generated by $\hat{A}$:

$$F(\hat{A} - A_{\text{prep.}}) = \int G(\phi) \, \text{exp} \left(-i \frac{1}{\hbar} (\hat{A} - A_{\text{prep.}}) \phi \right) d\phi$$

The filter function $F(A)$ is the Fourier transform of the phase shift distribution $G(\phi)$. The dynamics of control is limited by

$\delta A \delta \phi \geq \frac{\hbar}{2}$	Precision of control δA Dynamical randomization $\delta \phi$

For eigenstates of $\hat{A}$, ergodic randomization is complete ($\delta \phi \rightarrow \infty$).

Dynamics and quantum coherence

Complete ergodic randomization results in eigenstates of the property targeted by the procedure $\hat{F}(A_{\text{prep.}} = A_a)$ of state preparation:

$$\exp(-\frac{i}{\hbar}\hat{A}\phi) \mid A_a\rangle = \exp(-i\frac{A_a\phi}{\hbar}) \mid A_a\rangle$$

Eigenstates of other physical properties $\hat{H}$ represent different types of orbital motion:

$$\exp(-\frac{i}{\hbar}\hat{H}t) \mid E_n\rangle = \exp(-i\frac{E_n t}{\hbar}) \mid E_n\rangle$$

Quantum statistics is ergodic - it originates from the laws of motion and *not* from sets of realities.

"On the fundamental role of dynamics in quantum physics,"
H.F. Hofmann, Eur. Phys. J. D **70**, 118 (2016).

Ergodic randomization in quantum measurements

Measurements are described by filter operations $\widehat{F}(M_{\text{out}})$, where the resolution function $F(\Delta A)$ defines measurement precision,

$$F(\widehat{M} - M_{\text{out}}) = \int G(\tau) \exp\left(-i\frac{1}{\hbar}(\widehat{M} - M_{\text{out}})\tau\right) d\tau$$

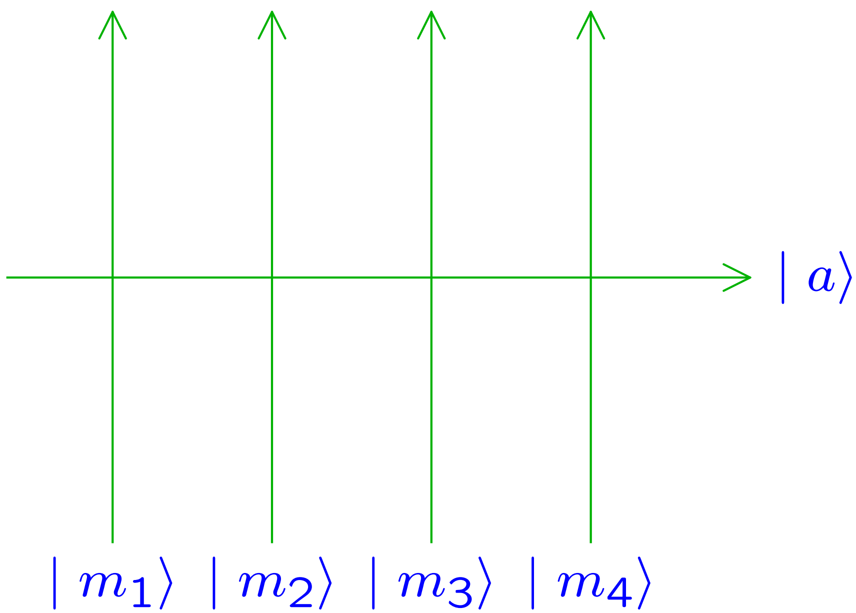
Precise measurements involve complete ergodic randomization of the dynamics generated by $\widehat{M}$ $(\delta\tau \to \infty)$.

$\Downarrow$

Decoherence is caused by the ergodic randomization of transformation distance τ along the orbit $\mid m\rangle$.

Quantum coherence as transformation distance

"Superpositions" describe intersections between orbits, where the phase localizes the intersection along the orbits.



Intersections:

$$\langle m \mid a \rangle$$

$$\Downarrow$$

$$\langle m \mid \hat{U}_M \mid a \rangle$$

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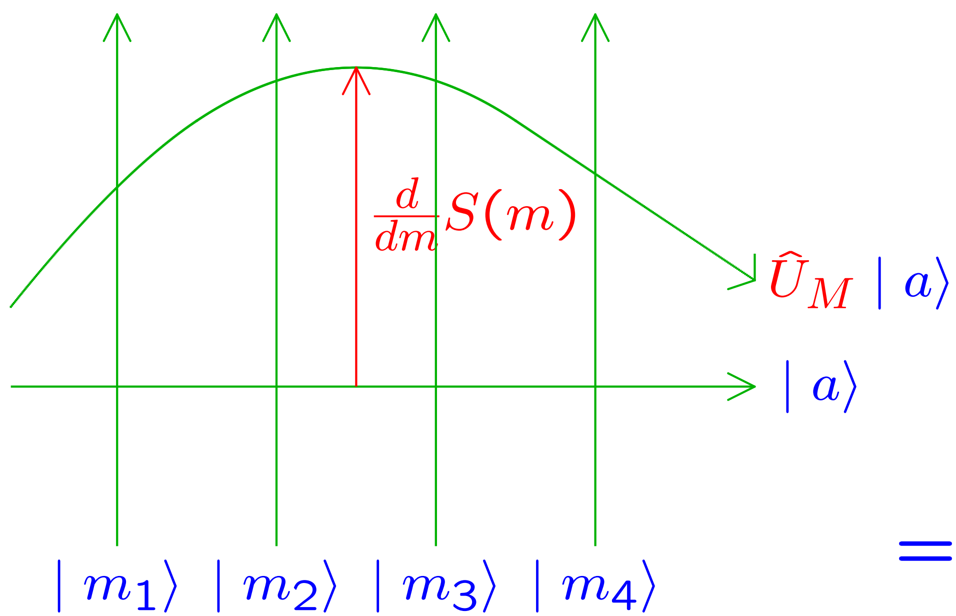


Intersections:

$$\langle m \mid a\rangle$$

$$\Downarrow$$

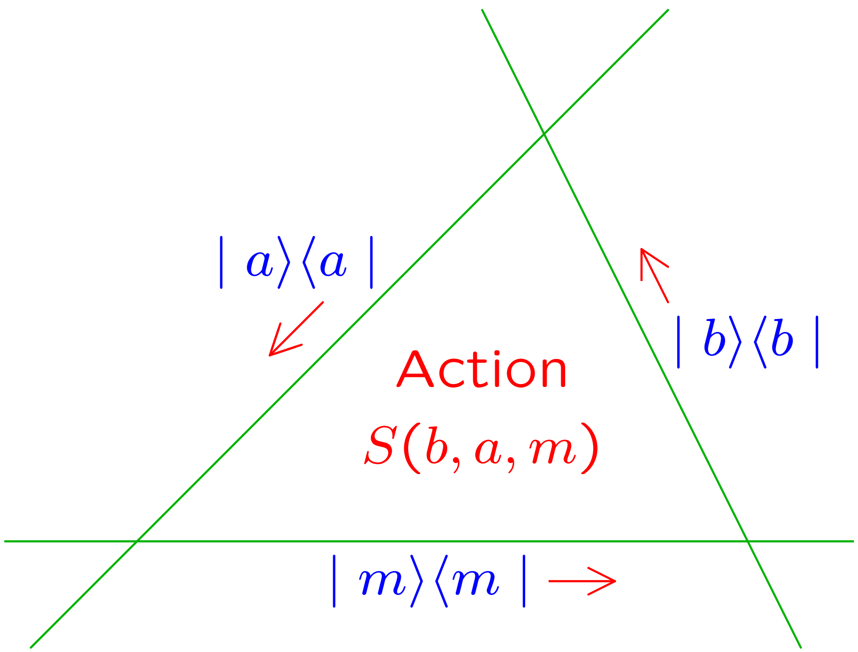
$$\langle m \mid \hat{U}_M \mid a\rangle$$

$$= \exp(-i\frac{S(m)}{\hbar})\langle m \mid a\rangle$$

Distances along m are described by gradients of *action phase* $S(m)$.

Relations between intersecting orbits: the dynamical origin of quantum coherence

Quantum states describe ergodic orbits. Complex inner products describe the *dynamics* of intersecting orbits.



Actions $S(b,a,m)$ describe transformations from a to b along m:

$$\text{Max}(|\langle b \mid \hat{U}_M \mid a\rangle|^2) \quad \text{for}$$

$$\hat{U}_M = \sum_m \mathrm{e}^{-i\frac{S(b,a,m)}{\hbar}} \, |m\rangle\langle m|$$

Weak measurements and Dirac distributions

Weak measurements describe the *deterministic relations* between three non-commuting properties as complex conditional probabilities,

$$p(m|a,b) = \frac{\langle b \mid m\rangle\langle m \mid a\rangle}{\langle b \mid a\rangle}$$

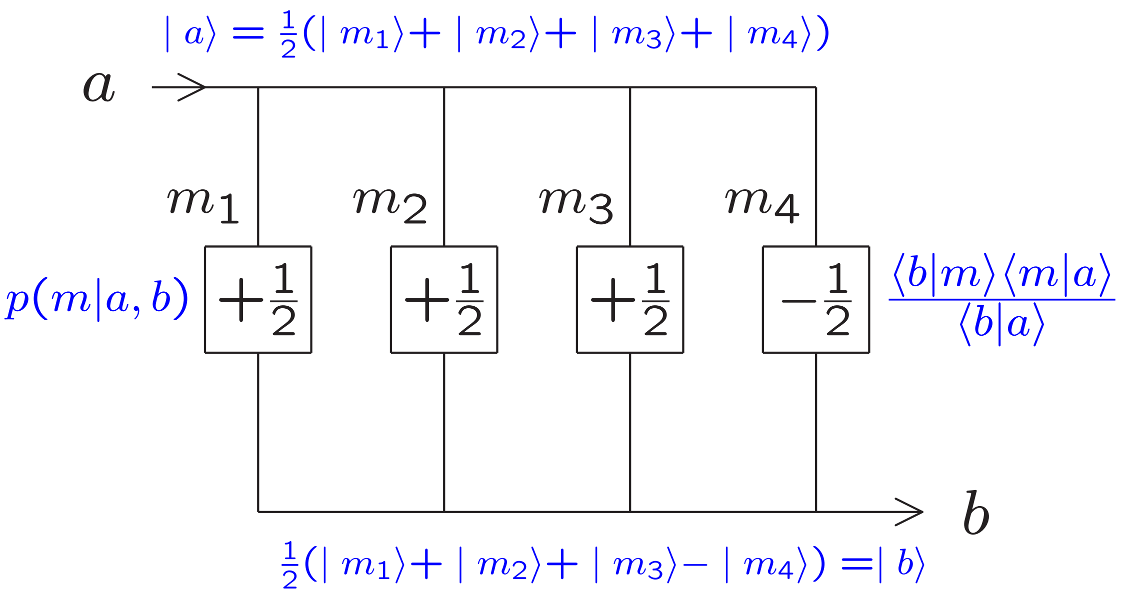
Using this relation, quantum states can be expressed by *complex joint probabilities*, as shown by Dirac in 1945:

$$\rho(a,b|\psi) = \langle b \mid a\rangle\langle a \mid \psi\rangle\langle \psi \mid b\rangle$$

Non-classical statistics result from the action phase $S(a,b,\psi)$ enclosed by a, b and ψ. (H. F. Hofmann, New J. Phys. **13**, 103009 (2011) and H. F. Hofmann, New J. Phys. **14**, 043031 (2012))

Quantum paradoxes and unitary transformations

Hofmann, Phys. Rev. A **91**, 062123 (2015)



$p(m_1) + p(m_2) = 1$
$p(m_3) + p(m_4) = 0$

OR

$p(m_1) + p(m_3) = 1$
$p(m_2) + p(m_4) = 0$

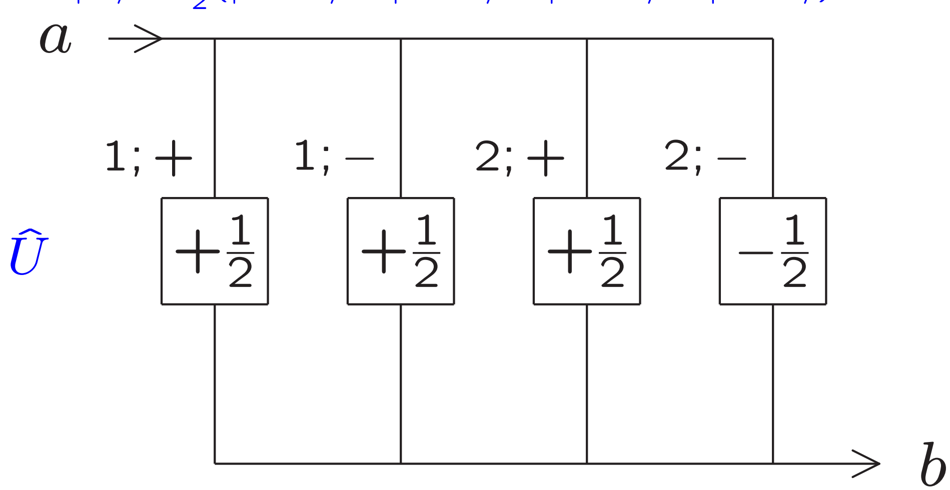
OR

$p(m_2) + p(m_3) = 1$
$p(m_1) + p(m_4) = 0$

Half-periodic transformations between a and b appear as negative conditional probabilities in the statistics of m.

Quantum Cheshire cat: Ergodic randomization and entanglement

$|a\rangle = \frac{1}{2}(|1;+\rangle + |1;-\rangle + |2;+\rangle + |2;-\rangle)$



$\frac{1}{2}(|1;+\rangle + |1;-\rangle + |2;+\rangle - |2;-\rangle) = |b\rangle$

"Cat":

$p(1;+) + p(1;-) = 1$

$p(2;+) + p(2;-) = 0$

"Smile":

$p(1;+) - p(1;-) = 0$

$p(2;+) - p(2;-) = 1$

Ergodic randomizations in a and in b are related by an entangling transformation between (1,2) and (+,-).

Relations between different contexts

Quantum correlations are defined by the action $S(m)$ of unitary transformations,

$$\langle b \mid \hat{U} \mid a \rangle = \sum_m P(m|a,b)\ \text{exp}\left(-i\frac{S(m)}{\hbar}\right)\langle b \mid a \rangle$$

All quantum interference effects can be traced to the action $S(m)$ of a reversible transformation $\hat{U}$.

$$\Downarrow$$

The macroscopic limit of transformations $\hat{U}$ can be used to derive the action-phases $S(m)$ of quantum interferences (Hibino et al., arXiv:1705.05118).

Quantum origin of macroscopic laws of motion

Quantum mechanics describes the process of motion from $\mid a\rangle$ to $\mid b\rangle$ within a time of t by

$$\langle b \mid \hat{U}(t) \mid a\rangle = \sum_n \langle b \mid n\rangle\langle n \mid a\rangle \ \exp\left(-i\frac{E_n t}{\hbar}\right)$$

The quantum coherence of $\langle b \mid n\rangle$ encodes the times of arrival t_ν at $\langle \hat{B}\rangle = B$ for energy expectation values of $\langle \hat{H}\rangle = E$,

$$\langle b \mid n\rangle = \sum_\nu A_\nu(B,E) \ \exp\left(i\frac{S_\nu(B,E)}{\hbar}\right); \quad t_\nu(B,E) = \frac{\partial}{\partial E} S_\nu(B,E)$$

As shown in Hibino et al., arXiv:1705.05118, different macroscopic arrival times $t_\nu(B,E)$ appear as quantum interference patterns in the energy spectrum of $\mid b\rangle$.

Quantum coherence as time-energy relation

The action $S(B,E)$ determines the time t_ν it takes to get from a $t=0$ "start line" to B at an energy of E,

$$t_\nu(B,E) = \frac{\partial}{\partial E} S(B,E)$$

The eigenstate $\mid m\rangle$ with $E=E_m$ can be expressed in the eigenstate basis $\mid b\rangle$ with $B=B_b$.

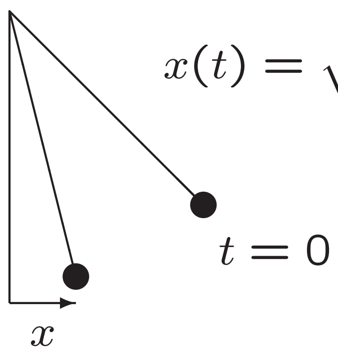
$$\langle b \mid m\rangle \approx \sqrt{\frac{\Delta B}{T_{\text{period}}}} \sum_\nu \sqrt{\frac{\partial}{\partial B} t_\nu(B,E)} \; \exp\left(i \frac{S(B,E)}{\hbar}\right)\bigg|_{B=B_b; E=E_m}$$

The amplitude is determined by the dwell time in the quantization interval ΔB, relative to the period of motion T_{period}.

Interferences between different arrival times

Quantum interference effects in the photon number distributions of a squeezed state explained by arrival time differences:

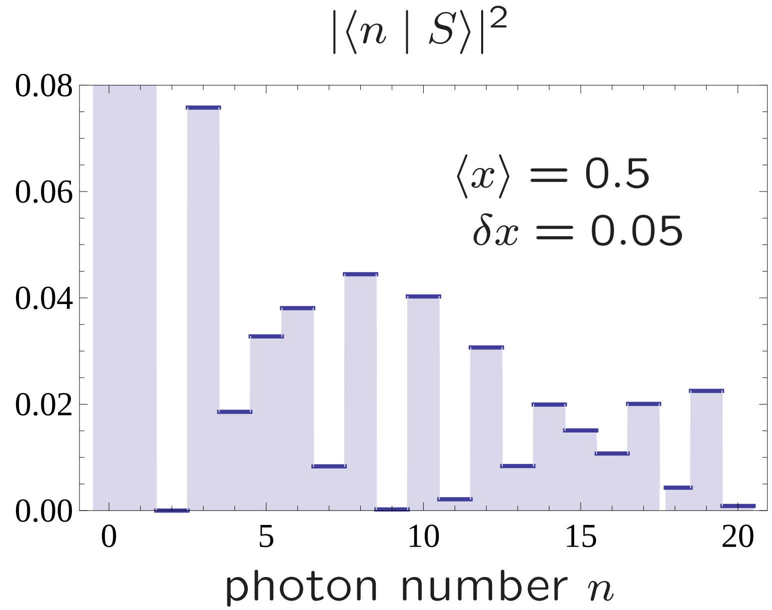
$$x(t) = \sqrt{n + 1/2}\,\cos(\omega t)$$



$$t_\nu(n) = \pm\left(\frac{\pi}{2\omega} - \frac{x}{\omega\sqrt{n+1/2}}\right)$$

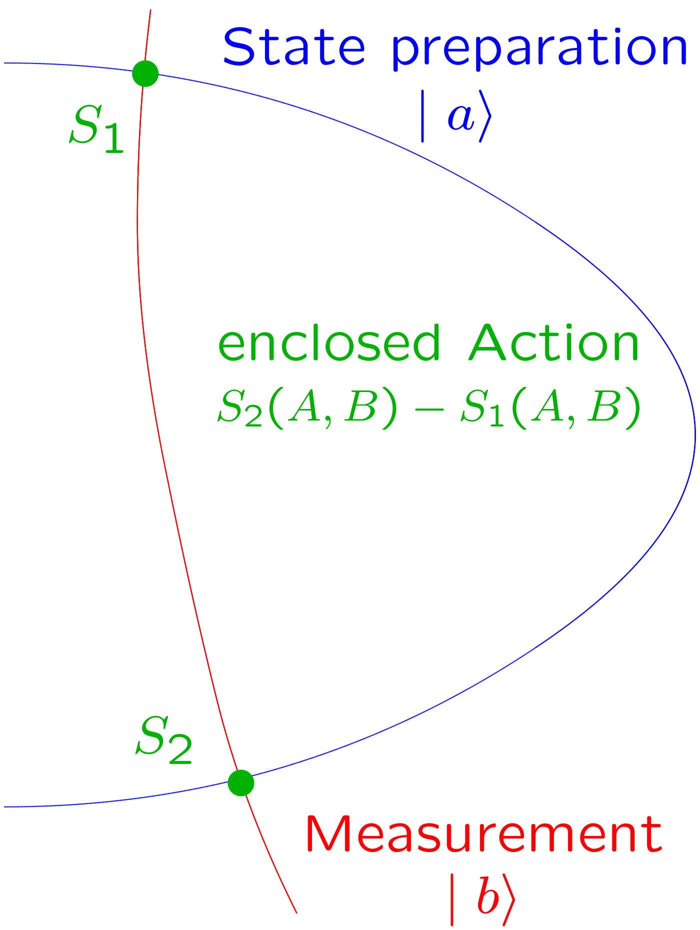
$$S_\nu(n) = \pm\hbar\left(\frac{\pi}{2}n - 2x\sqrt{n+1/2}\right)$$

$S_\nu(2) = 1.0\,\frac{\pi\hbar}{2}$; $S_\nu(9) = 7.0\,\frac{\pi\hbar}{2}$; $S_\nu(20) = 17.1\,\frac{\pi\hbar}{2}$



Field oscillation times successfully explain quantum interference patterns in the photon number distribution of a squeezed state.

Quantum causality



Without further interactions, physical causality relates the full initial orbit to the full final orbit.

$$\Downarrow$$

There are NO instantaneous or quasi-static realities in physics!

A Failure of Newtonian causality

Hofmann, Phys. Rev. A **96**, 020101(R) (2017).

A statistical limit for propagation along a straight line is given by

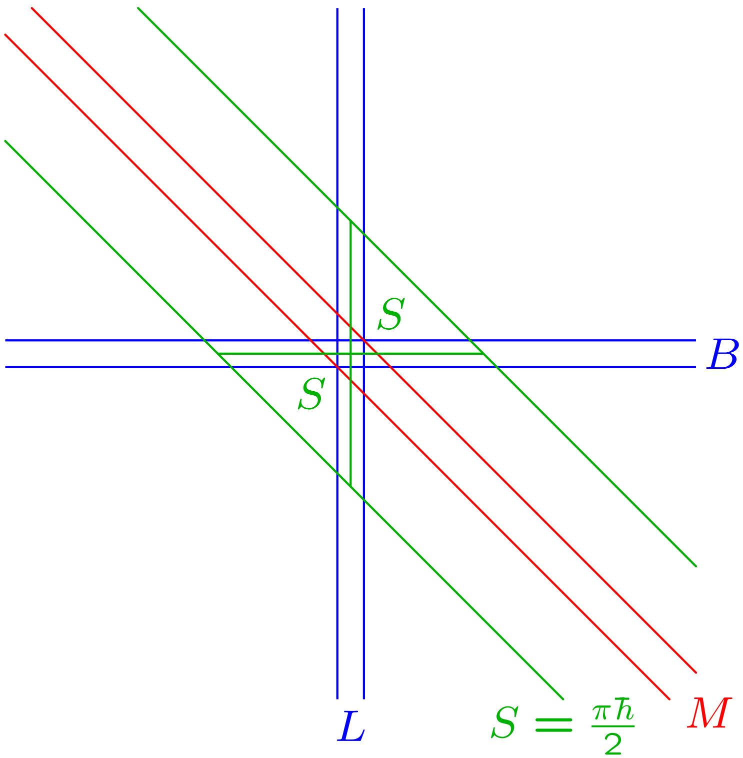
$$P\Big(\underbrace{|x(t)| \leq L}_{M}\Big) \geq P\Big(\underbrace{|x(0)| \leq \frac{L}{2}}_{L}\Big) + P\Big(\underbrace{\left|\frac{pt}{m}\right| \leq \frac{L}{2}}_{B}\Big) - 1$$

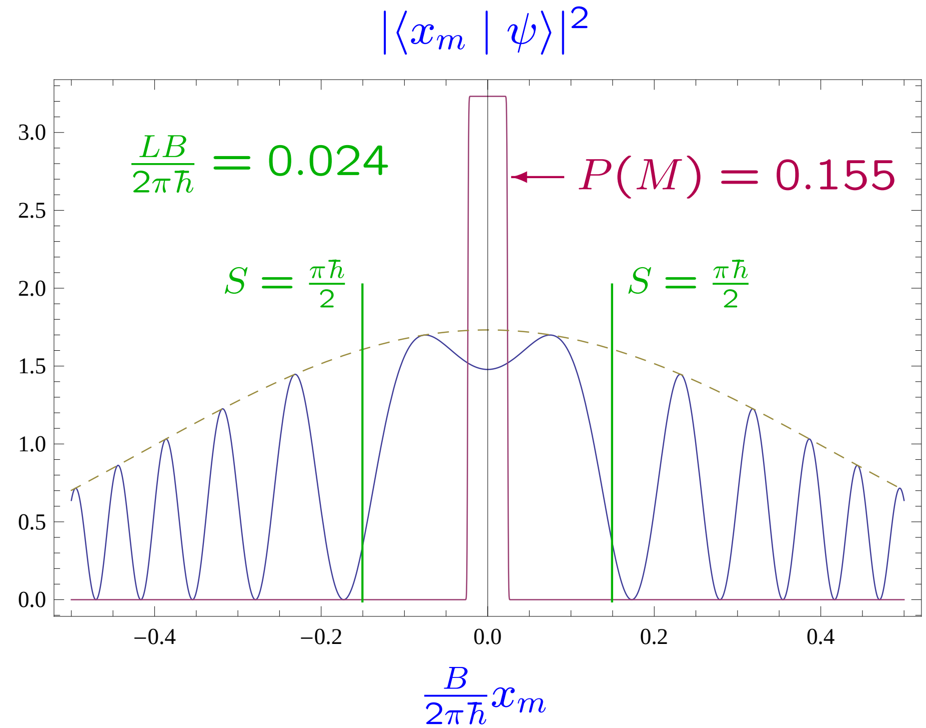
This inequality can be violated by superpositions of states localized in finite intervals of position and momentum,

$$\mid \psi\rangle = \frac{\mid L\rangle + \mid B\rangle}{\sqrt{2 + 2\langle L \mid B\rangle}}; \qquad P(L) + P(B) = 1 + \langle L \mid B\rangle$$

For $\langle L \mid B\rangle = 0.155$, the inequality is violated by $P(M) \approx 0.062$, which is about nine percent below the straight line limit.

Resolution of the particle propagation paradox





Enclosed action $S(x_m) = \frac{LB}{4}\left(\frac{x_m}{L}\right)^2$

Action phase causality of particle propagation

There is no joint reality of different positions $x(t)$. Operator relations correspond to complex conditionals with action phases,

$$\hat{x}(t) = \hat{x}(0) + \frac{\hat{p}}{m}t \quad \Rightarrow \quad P(x(t)|x(0),p) = \frac{\langle p \mid x(t)\rangle\langle x(t) \mid x(0)\rangle}{\langle p \mid x(0)\rangle}$$

The causality relation between $x(t)$, $x(0)$ and p is determined by the action phase enclosed by the three interecting orbits,

$$S(x(t),x(0),p) = \frac{t}{2m}\left(p - \frac{m}{t}(x(t) - x(0))\right)^2 - \frac{\pi\hbar}{8}$$

The conditional localization of $x(t)$ is achieved by quantum interference, limiting the localization to $\Delta x(t) = \sqrt{\hbar t/m}$.

Optimal control and deterministic probabilities

Quantum coherence emerges in a process of dynamic randomization. This process is required by the laws of nature that govern the causality of control.

1. Coherent randomizations can be expressed by superpositions, $\mid a\rangle = \sum_m \langle m \mid a\rangle \mid m\rangle$
2. Different measurements sample different dynamic relations: $\langle b \mid a\rangle = \sum_m \langle b \mid m\rangle\langle m \mid a\rangle$
3. Dynamic randomizations are related by action phases, $\langle b \mid \hat{U} \mid a\rangle = \sum_m \langle b \mid m\rangle\langle m \mid a\rangle \exp(-iS(m)/\hbar)$

Transformations fully determine quantum statistics. Macroscopically observable dynamics can predict and explain all quantum interference effects.

Outlook: quantum physics beyond paradoxes

- All causality in physics is described by action phases.

- Classical physics is explained as an approximation valid for low precision and limited control.

- There are no truly static realities. The action phase ratio $\hbar$ defines a finite amount of necessary dynamics.

A re-formulation of quantum mechanics based on dynamic relations can correct false intuitions. We need to work out a new intuitive understanding based on the empirically observed causality of natural processes.

More on the relation between quantum coherence and causality can be found in

1. "On the fundamental role of dynamics in quantum physics," H. F. Hofmann, Eur. Phys. J. D **70**, 118 (2016).

2. "Quantum effects in the interference of photon number states," H. F. Hofmann, K. Hibino, K. Fujiwara, and J.-Y. Wu, Phys. Rev. A **94**, 043809 (2016).

3. "Derivation of quantum statistics from the action of unitary dynamics," K. Hibino, K. Fujiwara, J.-Y. Wu, M. Iinuma, and H. F. Hofmann, e-print arXiv: 1705.05118

4. "Quantum interference of position and momentum: A particle propagation paradox," H. F. Hofmann, Phys. Rev. A **96**, 020101(R) (2017).

5. "Why interactions matter: How the laws of dynamics determine the shape of physical reality," H.F. Hofmann, June 2016 at the Perimeter Institute, Waterloo, http://pirsa.org/16060073

Quantum jumps and the limits of causal control

Quantum jumps are described by very weak unitary transformations, where initial state and final state are nearly orthogonal. Transition probablities are limited by energy uncertainties,

$$\frac{d}{dt} \mid a_1\rangle = -\frac{i}{\hbar}\hat{H} \mid a_1\rangle \quad \Rightarrow \quad P(a_2 \mid a_1; t) \leq \left(\frac{\Delta E\, t}{\hbar}\right)^2$$

Quantum jump events post-select extreme energy fluctuations,

$$P(a_2(t)|a_1(0), n) = \frac{\langle a_2 \mid n\rangle\langle n \mid a_1\rangle}{\langle a_2 \mid \hat{U}(t) \mid a_1\rangle} \exp(-\frac{i}{\hbar}E_n\, t)$$

Quantum jumps are a manifestation of extremal energy fluctuations in the initial state.