Measuring Coherence in Multislit Interference

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Measure of Coherence

PRL 113, 140401 (2014) PHYSICAL REVIEW LETTERS week ending 3 OCTOBER 2014

Quantifying Coherence

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(Received 24 February 2014; revised manuscript received 18 July 2014; published 29 September 2014)

l_1-norm of coherence:

$$C_{l_1}(\rho) = \sum_{j \neq k} |\rho_{jk}|$$

Shown to be a good measure of coherence
Minimum value is zero. Maximum value not fixed.

Coherence introduced as a theoretical quantifier

Can coherence be experimentally measured?

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Bohr's Complementarity Principle.

Niels Bohr in 1928
Certain physical concepts are complementary. If two concepts are complementary, an experiment that clearly illustrates one concept will obscure the other complementary one. . .

- An experiment that illustrates the particle properties of light will not show any of the wave properties of light.
- an experiment that illustrates the wave properties of light will not show any of the particle nature of light.

In the two-slit experiment, the **"which-way"** information and the existence of **interference** pattern are mutually exclusive.

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Either Wave Nature OR Particle Nature

Probing Wave & Particle natures together

What happens if one tries to observe both wave and particle nature at the same time?


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particles, fields, gravitation, and cosmology

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Complementarity in the double-slit experiment: Quantum nonseparability and a quantitative statement of Bohr's principle

William K. Wootters and Wojciech H. Zurek
Phys. Rev. D **19**, 473 – Published 15 January 1979

Trying to observe particle nature, blurs the interference

Quantitative Wave-Particle Duality

Duality relation for 2-slit interference [1]

$$\mathcal{D}^2 + \mathcal{V}^2 \leq 1$$

$\mathcal{D}$ $\rightarrow$ Path distinguishability
$\mathcal{V}$ $\rightarrow$ Visibility of interference

$$\mathcal{V} \equiv \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Going beyond two slits

$\mathcal{D}$ $\rightarrow$ no generalization to n-slits
$\mathcal{V}$ $\rightarrow$ may not be a convenient quantity for n-slits

[1] B-G. Englert, *Phys. Rev. Lett.* **77**, 2154 (1996).

Coherence as a measure of wave-nature

We introduce a quantity Coherence

$$\mathcal{C} \equiv \frac{1}{n-1} \sum_{j \neq k} |\rho_{jk}|$$

(is basis dependent)

Coherence values: $0 \leq \mathcal{C} \leq 1$.

For a maximally coherent state
$|\Psi\rangle = \frac{1}{\sqrt{n}} (|\psi_1\rangle + |\psi_2\rangle + |\psi_3\rangle + \cdots + |\psi_n\rangle)$

$$\mathcal{C} = 1$$

For a completely diagonal density matrix

$$\mathcal{C} = 0$$

Coherence can be a good measure of wave-nature

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Criteria for a good measure of wave-nature

S. Dürr, *Phys. Rev. A* **64**, 042113 (2001).

1. It should be measurable from the interference pattern, without referring to the matrix elements of ρ .
2. It should be a function of the matrix elements of ρ .
3. If the system shows no interference, it should reach its global minimum.
4. If ρ represents a pure state (i.e., $\rho^2 = \rho$) and all n beams are equally populated (i.e., all $\rho_{jj} = 1/n$), visibility should reach its global maximum.
5. Considered as a function in the parameter space $(\rho_{11}, \rho_{12}, \ldots, \rho_{nn})$, it should have only global extrema, no local ones.
6. It should be independent of our choice of the coordinate system.

Coherence $\mathcal{C}$ satisfies Dürr's criteria (2) through (6)

Wave-particle duality in n-path interference

Duality relation for n-slit interference [2]

$$\mathcal{D}_Q + \mathcal{C} \leq 1$$

Another duality relation for n-slit interference [3]

$$\mathcal{D}^2 + \mathcal{C}^2 \leq 1$$

$\mathcal{D}_Q \rightarrow$ Maximum probability with which one can unambiguously distinguish between the n paths

Visibility $\mathcal{V}$ can be easily measured in an experiment

Big question:
Can $\mathcal{C}$ be measured in an interference experiment?

[2] M.N. Bera, T. Qureshi, M.A. Siddiqui, A.K. Pati, Phys. Rev. A 92, 012118 (2015)

[3] T. Qureshi, M.A. Siddiqui, Ann. Phys. 385, 598-604 (2017)

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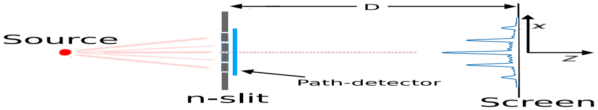

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Density matrix

$$\rho \equiv |\Psi\rangle\langle\Psi|$$

Reduced density matrix of the particle

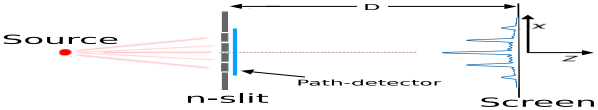
$$\rho_s \equiv Tr_{path-detector}|\Psi\rangle\langle\Psi| = \sum_{j=1}^{n}\sum_{k=1}^{n} c_j c_k^* \langle d_k|d_j\rangle\, |\psi_j\rangle\langle\psi_k|$$

Coherence

$$\mathcal{C} = \frac{1}{n-1}\sum_{j\neq k}|\langle\psi_j|\rho_s|\psi_k\rangle| = \frac{1}{n-1}\sum_{j\neq k}|c_j||c_k||\langle d_k|d_j\rangle|$$

n-slit (or n-path) interference

in the presence of a path-detector



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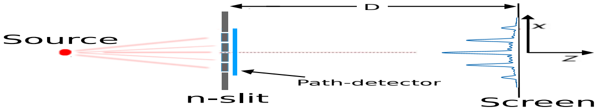

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n-path interference

Particle emerges from n slits

$$\begin{aligned}\langle x|\Psi(0)\rangle &= c_1\psi_1(x)|d_1\rangle + c_2\psi_2(x)|d_2\rangle + \cdots + c_n\psi_n(x)|d_n\rangle \\ &= A\sum_{j=1}^{n} c_j \exp\left(-\frac{(x-j\ell)^2}{\epsilon^2}\right)|d_j\rangle\end{aligned}$$

After a time t, the particle reaches the screen

$$\begin{aligned}|\langle x|\Psi(t)\rangle|^2 &= |A_t|^2\sum_{j=1}^{n}|c_j|^2\exp\left(-\frac{2\epsilon^2x^2}{(\lambda D/\pi)^2}\right) \\ &+ \sum_{j\neq k}|c_j||c_k||\langle d_j|d_k\rangle|\exp\left(-\frac{2\epsilon^2x^2}{(\lambda D/\pi)^2}\right) \\ &\times\ \cos\left(\frac{2\pi x l(k-j)}{\lambda D} + (\theta_k-\theta_j)\right).\end{aligned}$$

n-path interference continued . . .

Simplify: assume all the phases to be the same, i.e., $\theta_k - \theta_j = 0$.
For $x_m = m\lambda D/\ell,\ m = 0, 1, 2, \ldots,$
the cosine term is 1, irrespective of the values of j, k.
These are the positions of the primary maxima

Intensity at a primary maximum, $I_{max} = |\langle x_m|\Psi(t)\rangle|^2$:

$$I_{max} = |A_t|^2 \exp\left(-\frac{2\epsilon^2 x^2}{(\lambda D/\pi)^2}\right) \left[\sum_{j=1}^n |c_j|^2 + \sum_{j\neq k} |c_j||c_k||\langle d_j|d_k\rangle|\right]$$

If the same experiment is performed using incoherent light, one has to average over the phases $\theta_j, \theta_k \to$ cosine terms are killed!

The intensity, at the same position on the screen is

$$I_{inc} = |A_t|^2 \exp\left(-\frac{2\epsilon^2 x^2}{(\lambda D/\pi)^2}\right) \sum_{j=1}^n |c_j|^2$$

Experimentally measured coherence

Subtracting the previous two eqns, one can write

$$\frac{1}{n-1}\frac{I_{max}-I_{inc}}{I_{inc}} = \frac{1}{n-1}\sum_{j\neq k}|c_j||c_k||\langle d_j|d_k\rangle|$$

The RHS of the above is just the coherence $\mathcal{C}$!

Experimentally measured coherence

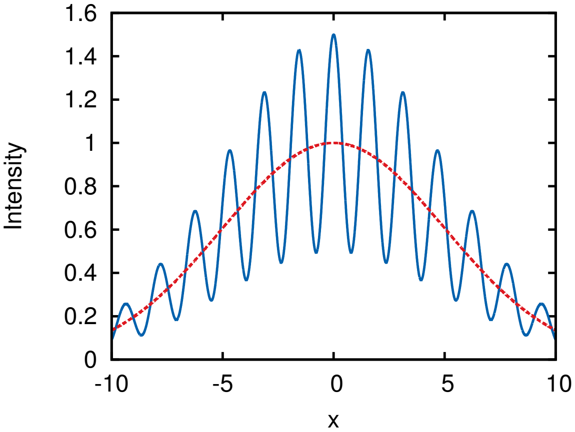
$$\mathcal{C}_{expt} = \frac{1}{n-1}\frac{I_{max}-I_{inc}}{I_{inc}}$$

Thus, $\mathcal{C}$ satisfies Dürr's criterion (1).

Same result holds for a mixed particle-path-detector state.

Coherence can be measured in a n-slit interference experiment

Coherence versus Visibility



$$\mathcal{V} \equiv \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

$$\mathcal{C}_{expt} = \frac{1}{n-1} \frac{I_{max} - I_{inc}}{I_{inc}}$$

Zero path knowledge case:
$|\langle d_j|d_k\rangle| = 1$ for all j, k, given by

$$I^{\parallel}_{max} = |A_t|^2 \exp\left(-\frac{2\epsilon^2 x^2}{(\lambda D/\pi)^2}\right)\left[\sum_{j=1}^{n}|c_j|^2 + \sum_{j\neq k}|c_j||c_k|\right].$$

Full path knowledge case:
$\langle d_j|d_k\rangle = 0$ for all $j \neq k$, given by

$$I^{\perp}_{max} = |A_t|^2 \exp\left(-\frac{2\epsilon^2 x^2}{(\lambda D/\pi)^2}\right)\sum_{j=1}^{n}|c_j|^2.$$

Coherence of the incoming quanton:

$$\mathcal{C}^0_{expt} = \frac{1}{n-1}\frac{I^{\parallel}_{max} - I^{\perp}_{max}}{I^{\perp}_{max}} = \frac{1}{n-1}\sum_{j\neq k}|c_j||c_k|$$

Coherence of the incoming particle

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Conclusions

- Coherence of a particle can be measured in a n-slit interference experiment.
- For N-slit interference, in the duality relations

$$\mathcal{D}_Q + \mathcal{C} \leq 1$$

$$\mathcal{D}^2 + \mathcal{C}^2 \leq 1$$

the wave-nature quantified by quantum *coherence* $\mathcal{C}$, now has an experimental meaning.

Measuring coherence in n-path interference
T.Paul, T. Qureshi, *Phys. Rev. A* 95, 042110 (2017).

Duality of quantum coherence and path distinguishability
M.N. Bera, T. Qureshi, M.A. Siddiqui, A.K. Pati, *Phys. Rev. A* 92, 012118 (2015).

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